

Graphic solution of R-W metric for the case of positive curvature and cosmic entire space and time

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Abstract

The vivid image of R-W metric for the case of hypersphere is given; The hypersphere is proposed as the only model for the physical universe; The cosmic radius $R(t)$ may be defined as a cosmic broad time and the cosmic entire space is the three-dimensional front (hypersphere) of $R(t)$ in four-dimensional Euclidean space; The structure of cosmic entire space and time is an absolute system.

Key words: R-W metric, hypersphere, cosmology, large scale structure of the universe, space and time, absolutism, relativism.

1. Introduction

Some authors in order to insist on the importance of the relationship between R-W metric [1] [2] and the Friedmann model, got misconception to call R-W metric as FRW metric. In fact, the R-W metric gotten is not only independent of Friedmann model but also general relativity. It is the cosmological principle that the metric based on only. Strictly speaking, the cosmological principle merely supposes that the cosmic entire space is homogeneous and isotropic. Its mathematic expression, as is well known, can be written as:

$$\begin{aligned} ds^2 &= R^2(t)[d\alpha^2 + \sin^2\alpha(d\theta^2 + \sin^2\theta d\varphi^2)]; & 0 \leq \alpha \leq 2\pi; & \text{for } k=+1, \text{ spherical space;} \\ ds^2 &= R^2(t)[d\alpha^2 + \alpha^2(d\theta^2 + \sin^2\theta d\varphi^2)]; & 0 \leq \alpha < \infty; & \text{for } k=0, \text{ flat space;} \\ ds^2 &= R^2(t)[d\alpha^2 + \sinh^2\alpha(d\theta^2 + \sin^2\theta d\varphi^2)]; & 0 \leq \alpha < \infty; & \text{for } k=-1, \text{ hyperbolic space.} \end{aligned}$$

According to cocker, the space of the universe observed at the moment is expanding with the growing of $R(t)$ or time t . In respect that the entire space is an emblematical instantaneous space, it is necessary for us to point up the concept of synchronism first. As we know that proper time is already used to describe for entire universe on the point of view of a fundamental observer. in order to dig the denotative and connotative meanings from the principle, we may give the relationship between cosmic entire space and proper time as follows:

(a) The all matter in cosmic space has the same history at any given proper time, it means that the cosmic age is same on isochronous three-dimensional hypersurface space, and the matter has been experiencing the same historical course of evolvement;

(b) The cosmic large-scale space and time remain orthogonal relation with each other everywhere in the cosmic entire space.

The two items above imply that the cosmic entire space and proper time are symmetric to an origin in four-dimensional Euclidean space. As for the two case of both flat and hyperbolic space,

there aren't any real significance for $R(t)$, it just be cursorily called "cosmic scale factor" and can't be defined. Whereas, the spherical space is quite different from them, it is not only compact but also can give a clear physical significance to $R(t)$. As we argued before (Deng Xiaoming 2005a) [3], in four-dimensional Euclidean space, vector $R(t)$ can be defined as "timelike" function.

In fact, the cosmological principle is not a phantasy, it found on abundant observations. According to the lens effect of cosmic entire hyperspherical space (Deng Xiaoming 2005b) [4], as long as the cosmic prospect observed by an ordinary observer (like us) is spherical symmetry to him, the cosmological principle is tenable. The basic operation of the principle is to regulate the modality of cosmic entire space and time despite the fact that we don't like divisibility between space-time and matter. Of course, we have noticed that whenever the cosmic space and time are defined, it can't be separate from matter in philosophy. In a word, any theory is not indubitability in cosmology today, all of hypothesis or theory (no matter is standard or not) just like working ladder to help us to reach the fact, only observational data can be the criterion of truth.

2. Graphic solution of R-W metric for the case of spherical space

With the help of Fig 1, it is very easy to give or show R-W metric for the case of spherical space. In four-dimensional Euclidean space, set up an absolute cartesian reference frame, the transformation with hyperspherical coordinates are:

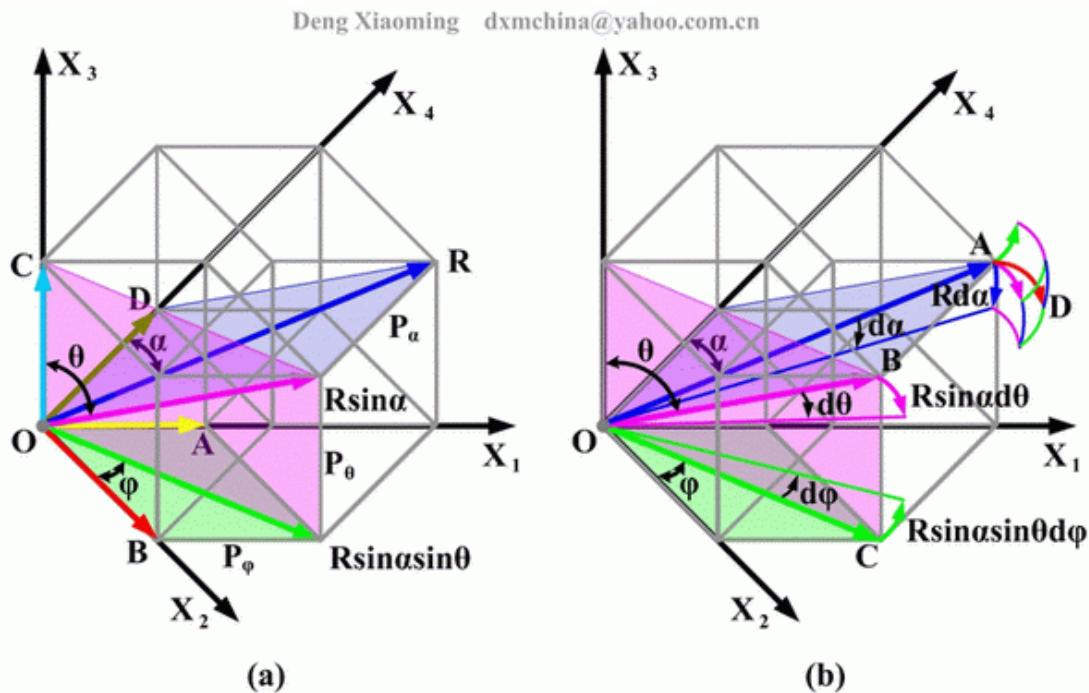


Figure 1 The hyperspherical coordinates in four-dimensional Euclidean space

(Reference Fig 1 (a), vector R , $R \sin \alpha$ and $R \sin \alpha \sin \theta$ are on mutually perpendicular plane P_α , P_θ and P_ϕ respectively).

$$X_1 = OA = R \sin \alpha \sin \theta \sin \phi; X_2 = OB = R \sin \alpha \sin \theta \cos \phi; X_3 = OC = R \sin \alpha \cos \theta; X_4 = OD = R \cos \alpha$$

$$\text{We also have: } R^2 = X_1^2 + X_2^2 + X_3^2 + X_4^2 = (OA)^2 + (OB)^2 + (OC)^2 + (OD)^2$$

Reference Fig 1 (b), on plane P_α , P_θ and P_φ , if angle increment of vector R , $R\sin\alpha$ and $R\sin\alpha\sin\theta$ are $d\alpha$, $d\theta$ and $d\varphi$ respectively, the corresponding arc length are $Rd\alpha$, $R\sin\alpha d\theta$ and $R\sin\alpha\sin\theta d\varphi$, then at the end of vector R , we have

$$dS^2=ds^2=(Rd\alpha)^2+(R\sin\alpha d\theta)^2+(R\sin\alpha\sin\theta d\varphi)^2=R^2[d\alpha^2+\sin^2\alpha(d\theta^2+\sin^2\theta d\varphi^2)]$$

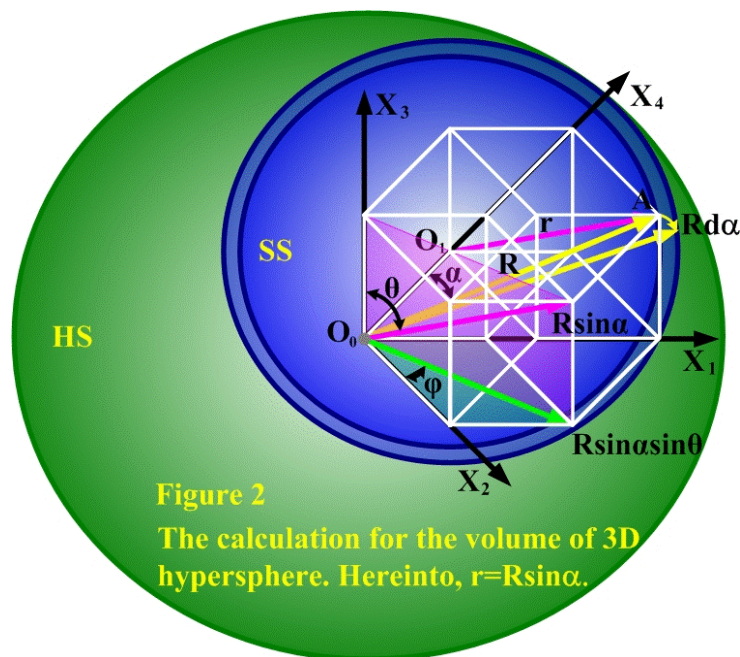
Here we use very simple and visual way to get R-W metric for the case of spherical space. We also can get the vixel of hypersphere at the end of vector R :

$$dV=(Rd\alpha)(R\sin\alpha d\theta)(R\sin\alpha\sin\theta d\varphi)=R^3\sin^2\alpha\sin\theta d\alpha d\theta d\varphi, \text{ and the volume of hypersphere is}$$

$$V=R^3 \int_0^\pi \sin^2\alpha d\alpha \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\varphi = 2\pi^2 R^3.$$

According to Fig 2, We also can get the volume of hypersphere in other way. Since $O_1A=r=R\sin\alpha$, the surface area of sphere with radius $r=R\sin\alpha$ is: $4\pi r^2=4\pi(R\sin\alpha)^2$, and the thickness of the spherical shell SS is $Rd\alpha$, the vixel of this spherical shell is: $dV=4\pi(R\sin\alpha)^2 Rd\alpha$, then the volume of the hypersphere HS when R is fixed is:

$$V=4\pi R^3 \int_0^\pi \sin^2\alpha d\alpha = 2\pi^2 R^3.$$



Where O_0 and O_1 are the origins of HS and SS respectively, and R is the radius of hypersphere HS . From this integral process, we also can partly understand the property of hypersphere intuitively.

See also Fig 1, If let $\alpha=\pi/2$, the three-dimensional hypersphere will degenerate into two-dimensional sphere under coordinate axis (X_1, X_2, X_3) : $ds^2=R^2(d\theta^2+\sin^2\theta d\varphi^2)$; If let $\theta=\pi/2$, the two-dimensional sphere under coordinate axis (X_1, X_2, X_4) is given as: $ds^2=R^2(d\alpha^2+\sin^2\alpha d\varphi^2)$; analogously, if let $\alpha=\theta=\pi/2$, elementary arc: $ds=Rd\varphi$ is given...

In a word, with the help of Fig 1 and Fig 2, it is very easy (as same as a high school geometrical

problem) for us to discuss many geometrical characteristic of hypersphere visually. All of this may help us to explain the profound kernel of our universe in a simple way.

3. The proposed structure of cosmic entire space and time

We have already pointed out that General Relativity can not be dictator in cosmology at all [3]. Scientific approach and academic freedom allow us to set up the possible structure of cosmic entire space and time. The main thing is to correspond with observations.

There are ready frame members in deed for us to set up the possible structure of cosmic entire space and time, that is the hypersphere in Riemannian geometry. Actually, as early as the year 1854, Riemann himself proposed the hypersphere as a model for the physical universe too. The key to the model is how to give idiographic physical significance to geometrical elements.

As we know that $R(t)$ is the most important function, definitely speaking, it is the very core of cosmology. To study function $R(t)$, any inexplicit concept can't be allowed. As we mentioned above, $R(t)$ can be strictly defined in the case of spherical space only, so we call it as cosmic radius.

We have considered that cosmic radius $R(t)$ is a kind of vector with the character of cosmic time (Deng Xiaoming 2005a) [3]. In four-dimensional Euclidean space, vector $R(t)$ can be defined as a "timelike" function, or as a matter of convenience, we might as well call it as "a cosmic broad time". The cosmic broad time is endowed with all of the characteristic of time but the measure of length, and its direction is outgoing hyperspherically symmetric radiation from the origin in four-dimensional Euclidian space. Hereby, we can define the cosmic entire space as the front of the cosmic broad time, that is the three-dimensional hyperspherical space: $2\pi^2R^3(t)$. Note that we still continue to have the concept of proper time t here. This definition accord with our hypothesis (a) and (b) in section 1, and it is rational because $R(t)$ is not a ordinary spatial element, it is not in the interior of cosmic entire space.

We have given the clear structure of cosmic entire space and time above. It's worth noting that, not only according to the physical significance of the cosmic geometry but also the R-W metric itself, the cosmic radius or the cosmic broad time $R(t)$ is a self-evident absolute coordinate. It can mark cosmic evolution, cosmic spatial curvature and the absolute value of redshift and so on. Of course, it is reasonless to go from the sublime to the ridiculous, there is not only absolutism but also relativism in the universe, and the nub of the problem is the range of application for each.

As for the determination of $R(t)$, we also argued before [3], and we will penetratingly prove that $R(t)$ can't be determined by any gravitation theory in next paper.

4. Discussion

it is necessary to notice that there seem to be two kinds of observers in the universe: the one are the ordinary observers in three-dimensional hyperspherical space, like us on a planet at the same proper time or the cosmic broad time; another is "the fundamental observer" in four-dimensional Euclidian space (maybe mentioned by H. P. Robertson first [1]), and for all we know, the alleged fundamental observer is absolute observer indeed, they can use the devices calibrated by proper standard, such as clocks, theodolites and the apparatus for sending and receiving light signals, to map the universe, and give the absolute coordinates α , θ , φ , $R(t)$ to locate an astronomical event.

In fact, the scenes of the real universe presented to each observer's eye are quite different [4], the former is the real observer, who can see some astronomical phenomena and periodicity caused by

the lens effect of cosmic entire space [4], and the latter is the observer imagined by us, who can “see” nothing different with the scene described by the cosmological principle and think what kind of universe should be based on the knowledge gained. Some authors can not distinguish between the two, and to the extent that the cosmological principle would be negated when some periodicity emerge in observations.

5. Conclusion

- (a) R-W metric gotten is not only independent of Friedmann model but also general relativity;
- (b) The hypersphere is proposed as the only model for the physical universe;
- (c) The cosmic radius $R(t)$ may be defined as a cosmic broad time and the cosmic entire space is the three-dimensional front (hypersphere) of $R(t)$ in four-dimensional Euclidean space;
- (d) The structure of cosmic entire space and time is an absolute system.

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